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Higher Order Squeezed States of Anharmonic Oscillators

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In this paper the N eigenstates of the Nth powers ($N \geq 3$) of the annihilation operator of an anharmonic oscillator are constructed and their completeness is investigated. We introduce a new kind of higher order squeezing, Mth-order $SU(1, 1)$ squeezing. The properties of Mth-order $SU(1, 1)$ squeezing and anticorrelation of the N states are studied. The result show that these states may form a complete Hilbert space; the Mth-order $[M = (n + 1/2)N;\ n = 0,1, \ldots]$ $SU(1, 1)$ squeezing effects exist in all of the N states when N is even. There is anticorrelation in all of them.

1. INTRODUCTION

Glauber [1] introduced coherent states into quantum optics and thereby solved the mathematical difficulties which had been met in the study of light using quantum electrodynamics, and greatly promoted the development of quantum optics. It is well known that coherent states are eigenstates of the annihilation operator a of a harmonic oscillator. The theory of coherent states and their application has become an important field [2] because coherent states not only have physical substance, but also construct a very useful representation. It is also noteworthy that eigenstates of the square a^2 of the annihilation operator of the harmonic oscillator, which are called even and odd coherent states [3], respectively, show two kinds of nonclassical effects; the even coherent state shows the squeezing effect, but no antibunching, while the odd coherent state shows the antibunching effect, but no squeezing.

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In our previous papers [4–6], the eigenstates of higher powers of the annihilation operator a of the harmonic oscillator were constructed and their nonclassical properties were investigated.

In fact, many realistic problems depart from the ideal harmonic oscillator model. Therefore, the study of anharmonic oscillators is significant. Recently, the concept of generalized coherent states was generalized to an anharmonic oscillator potential; they are the eigenstates of the annihilation operator A_- of an anharmonic oscillator. The generalized even and odd coherent states are also presented by Xu [7], and are the eigenstates of the square A_-^2 of the annihilation operator of the anharmonic oscillator; their squeezing properties were investigated [7, 8]. At this stage a natural question arises: What are the eigenstates of the higher powers of the annihilation operator for this anharmonic oscillator? Are they classical or nonclassical? In this paper, we find the eigenstates of higher powers A_-^N ($N \geq 3$) of the annihilation operator for this anharmonic oscillator. We introduce a new kind of higher order squeezing, higher-order $SU(1, 1)$ squeezing. The completeness, higher order $SU(1, 1)$ squeezing properties, and anticorrelation of the states are discussed.

2. EIGENSTATES AND THE MATHEMATICAL PROPERTIES OF THE OPERATOR A_-^N

For convenience of reference and completeness, in this section we begin with some related results for the anharmonic oscillator [7–10].

The Hamiltonian of an anharmonic oscillator is [10]

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 + \frac{1}{2}\frac{g}{x^2}, \qquad g > 0 \tag{1}$$

where $m = \hbar = \omega = 1$. Let $g = l(l + 1)$; from (1) we have

$$H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 + \frac{1}{2}\frac{l(l+1)}{x^2} \tag{2}$$

To ensure $g > 0$, we choose $l < -1$ and $l = -1/2 - \sqrt{g + 1/4}$ [9]. According to ref. 11, the corresponding natural coordinate operator Q and natural momentum operator P are, respectively,

$$Q = x^2 - H, \qquad P = \frac{1}{2i}\left(x\frac{d}{dx} + \frac{d}{dx}x\right) \tag{3}$$

They obey the commutation relations

$$[H, Q] = -2iP, \qquad [H, P] = 2iQ, \qquad [Q, P] = 2iH \tag{4}$$

For this anharmonic oscillator, we define the creation and annihilation operators as [7–10]

$$A_+ = \frac{1}{2}(Q - iP), \qquad A_- = \frac{1}{2}(Q + iP) \tag{5}$$

Therefore

$$[H, A_\pm] = \pm 2A_\pm, \qquad [A_-, A_+] = H \tag{6}$$

Let $H = 2A_0$; substituting it into (6), we find that the operators $A_\pm$ and A_0 can form a specific bosonic representation of the $SU(1, 1)$ Lie algebra; i.e., $A_\pm$ and A_0 are the generators of the $SU(1, 1)$ Lie group [12]. Assuming that $|n; k\rangle$ is the nth energy eigenstate of this anharmonic oscillator, we have [7–9, 12]

$$H|n; k\rangle = 2(n + k)\, |n; k\rangle \tag{7a}$$

$$A_+|n; k\rangle = \sqrt{(n + 1)(n + 2k)}\, |n + 1; k\rangle \tag{7b}$$

$$A_-|n; k\rangle = \sqrt{n\,(n + 2k - 1)}|n - 1; k\rangle \tag{7c}$$

where $k = (1 - \sqrt{g + 1/4})/2$ [9] and $A_-|0; k\rangle = 0$ [12]. From (7), we obtain

$$|n; k\rangle = \frac{1}{\sqrt{n!(2k)_n}}\, b_+^n|0; k\rangle \tag{8}$$

with $(2k)_n = (2k)(2k + 1) \cdots (2k + n - 1) = \Gamma(n + 2k)/\Gamma(2k)$.

2.1. Eigenstates of the Operator A_-^N

Following the definition of the generalized even and odd coherent states [7, 8] (i.e., the eigenstates of the operator A_-^2), we consider the following N states ($N \geq 3$):

$$|\psi_j\rangle_N = C_j \sum_{m=0}^{\infty} \frac{\beta^{mN+j}}{\sqrt{(mN + j)!(2k)_{mN+j}}}\, |mN + j; k\rangle$$
$$(j = 0, 1, 2, \ldots, N - 1) \tag{9}$$

where β is a complex number. The C_j ($j = 0, 1, 2, \ldots, N - 1$) are normalization factors, which can be obtained by letting ${}_N\langle\psi_j|\psi_j\rangle_N = 1$ ($j = 0, 1, 2, \ldots, N - 1$). Thus we have

$$\begin{aligned} C_j(r^2) &= [B_j(r^2)]^{-1/2} \\ &= \left[\sum_{m=0}^{\infty} \frac{r^{2(mN+j)}}{(mN + j)!(2k)_{mN+j}}\right]^{-1/2} \qquad (j = 0, 1, 2, \ldots, N - 1) \end{aligned} \tag{10}$$

where $r = |\beta|$. It is easy to prove that the N states of (9) are all eigenstates of operator A^N ($N \geq 3$) with the same eigenvalue β^N, and they are orthogonal to each other, i.e.,

$$A^N|\psi_j\rangle_N = \beta^N|\psi_j\rangle_N, \qquad {}_N\langle\psi_i|\psi_j\rangle_N = 0 \tag{11}$$
$$(i, j = 0, 1, 2, \ldots, N-1;\ i \neq j)$$

For a fixed value of N, it is obvious that the N states given by (9) are formally similar to the Barut–Girardello coherent state [12, 13]. In particular, for $N = 1$ ($j = N - 1 = 0$), the state $|\psi_0\rangle_1$ is just about the Barut–Girardello coherent state or the $SU(1, 1)$ intelligent state [14].

2.2. Mathematical Properties of the Eigenstates of A_-^N

First, it is seen that the eigenstates of the operator A_-^N contain the complex parameter β. When β takes different values, the internal product of every eigenstate does not equal zero, i.e.,

$$\begin{aligned} {}_N\langle\psi_j(\beta)|\psi_j(\beta')\rangle_N &= [B_j(|\beta|^2)B_j(|\beta'|^2)]^{-1/2} \sum_{m=0}^{\infty} \frac{(\beta^*\beta')^{mN+j}}{(mN+j)!(2k)_{mN+j}} \\ &= [B_j(|\beta|^2)B_j\,(|\beta'|^2)]^{-1/2}\, B_j(\beta^*\beta') \neq 0 \quad (\text{if} \quad \beta \neq \beta') \end{aligned} \tag{12}$$

This means that, in the β manifold, each of the N eigenstates of the operator A_-^N is not orthogonal by itself. This property is the same as that of the normal coherent states.

Second, in the space consisting of the N eigenstates of the operator A_-^N, each of the N eigenstates can be generated by the annihilation operator A_-. For example, if the operator A_- is used successively on $|\psi_0\rangle_N$, we get

$$A_-^i|\psi_0\rangle_N = \beta^i B_0^{-1/2} B_{N-i}^{1/2}|\psi_{N-i}\rangle_N \qquad (i = 1, 2, \ldots, N) \tag{13}$$

That is, under the action of A_-, the eigenstate $|\psi_0\rangle_N$ may be transformed in turn as follows: $|\psi_0\rangle_N \to |\psi_{N-1}\rangle_N \to |\psi_{N-2}\rangle_N \to \cdots |\psi_1\rangle_N \to |\psi_0\rangle_N$. Therefore, the operator A_- plays the role of a 'rotation operator' among the N eigenstates of the operator A_-^N.

The final question that concerns us is whether the N states given by (9) could construct a complete Hilbert space, i.e., whether they could be used as a representation. In order to construct the completeness formula for the N states, we use the density operator method [15]. We define the density operator (i.e., a density matrix) of the state $|mN + j; k\rangle$

$$\rho_j = \sum_{m=0}^{\infty} P(mN + j; k)|mN + j; k\rangle\langle mN + j; k| \tag{14}$$

where $P(mN + j; k) = \int P(mN + j, \beta; k)\, d^2\beta$ is the probability distribution of the $(mN + j)$th energy eigenstate $|mN + j; k\rangle$ of the anharmonic oscillator appearing in the state $|\psi_j\rangle_N$ in which

$$P(mN + j, \beta; k) = |\langle mN + j; k|\psi_j\rangle_N|^2 = \frac{1}{B_j(|\beta|^2)} \frac{|\beta|^{2(mN+j)}}{(mN + j)!(2k)_{mN+j}} \tag{15}$$

Thus we have

$$\rho_j^{-1} = \sum_{m=0}^{\infty} P^{-1}(mN + j; k)|mN + j; k\rangle\langle mN + j; k|$$

Therefore, the completeness formula of the N states given by (9) can be written as

$$\sum_{j=0}^{N-1} \rho_j^{-1} \int d^2 \beta|\psi_j\rangle_N \cdot {}_N\langle\psi_j| = 1 \tag{16}$$

The proof of the Eq. (16) is given as following:

$$\begin{aligned}
&\sum_{j=0}^{N-1} \rho_j^{-1} \int d^2\beta \, |\psi_j\rangle_N \cdot {}_N\langle\psi_j| \\
&\quad = \sum_{j=0}^{N-1} \rho_j^{-1} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{\sqrt{(mN + j)!(2k)_{mN+j}(nN + j)!(2k)_{nN+j}}} \\
&\qquad \times \int d^2\beta \, \frac{\beta^{mN+j}\beta^{*(nN+j)}}{B_j(r^2)} \, |mN + j; k\rangle\langle nN + j; k| \\
&\quad = \sum_{j=0}^{N-1} \rho_j^{-1} \sum_{n=0}^{\infty} 2\pi \int r\, dr \, \frac{(r^2)^{nN+j}}{B_j(r^2)(nN + j)!(2k)_{nN+j}} \, |nN + j; k\rangle\langle nN + j; k| \\
&\quad = \sum_{j=0}^{N-1} \rho_j^{-1} \sum_{n=0}^{\infty} P(nN + j; k)| \, nN + j; k\rangle\langle nN + j; k| \\
&\quad = \sum_{j=0}^{N-1} \sum_{m=0}^{\infty} P^{-1}(mN + j; k)|mN + j; k\rangle\langle mN + j; k| \\
&\qquad \times \sum_{n=0}^{\infty} P(nN + j; k)| \, nN + j; k\rangle\langle nN + j; k| \\
&\quad = \sum_{n=0}^{\infty} |n; k\rangle\langle n; k| = 1
\end{aligned} \tag{17}$$

where $\beta = r \exp(i\theta)$ and $d^2\beta = r\, dr\, d\theta$. Therefore, the linear combination of the N states may form a complete representation. For example, in this representation, the generalized coherent states $|\beta\rangle$ [7] of the anharmonic oscillator may be expressed as

$$|\beta\rangle = [F(|\beta|^2)]^{-1/2} \sum_{j=0}^{N-1} B_j^{1/2}(|\beta|^2)|\psi_j\rangle_N, \qquad F(|\beta|^2) = \sum_{n=0}^{\infty} \frac{|\beta|^{2n}}{n!(2k)_n} \tag{18}$$

3. HIGHER ORDER *SU*(1, 1) SQUEEZING OF THE EIGENSTATES OF THE OPERATOR A_-^N

3.1. Definition of Higher Order *SU*(1, 1) Squeezing

In analogy to the definition of higher order squeezing for the conventional single mode of the electromagnetic field [16], we define two Hermite operators

$$W_1(M) = (A_+^M + A_-^M)/2, \qquad W_2(M) = i(A_+^M - A_-^M)/2 \tag{19}$$

It can be proved that the operators $W_1(M)$ and $W_2(M)$ satisfy the commutation relation

$$[W_1(M), W_2(M)] = (i/2)[A_-^M, A_+^M] \tag{20}$$

and the uncertainty relation

$$\langle(\Delta W_1)^2\rangle \cdot \langle(\Delta W_2)^2\rangle \geq \frac{1}{16} |\langle[A_-^M, A_+^M]\rangle|^2 \tag{21}$$

A state is squeezed to order M if

$$\langle(\Delta W_1)^2\rangle - \frac{1}{4} \langle[A_-^M, A_+^M]\rangle < 0, \qquad (i = 1, 2) \tag{22}$$

From (19) and (22), we can see that it is *SU*(1, 1) squeezing [14] when $M = 1$. Therefore, this kind of higher order squeezing is a natural generalization of *SU*(1, 1) squeezing. We call it higher order *SU*(1, 1) squeezing. It is formally similar to the higher order squeezing defined by Zhang *et al.* [16].

3.2. Properties of Higher Order *SU*(1, 1) Squeezing of the Eigenstates of Operator A_-^N

Now we study the properties of higher order *SU*(1, 1) squeezing for the N states given by (9) in the following four cases.

3.2.1. $M = nN$ ($n = 1, 2, 3, \ldots$) for Even and Odd N

In this situation, for all of the states given by (9), we have from (11)

$${}_N\langle\psi_j|A_+^{2M}|\psi_j\rangle_N = r^{2nN} e^{-i2nN\theta}, \qquad {}_N\langle\psi_j\, |A^{2M}|\psi_j\rangle_N = r^{2nN} e^{i2nN\theta} \tag{23a}$$

$${}_N\langle\psi_j|A_+^{M}|\psi_j\rangle_N = r^{nN} e^{-inN\theta} \qquad {}_N\langle\psi_j|A^{M}|\psi_j\rangle_N = r^{nN} e^{inN\theta} \tag{23b}$$

$$ {}_N\langle\psi_j|A_+^M A_-^M|\psi_j\rangle_N = r^{2nN} \tag{23c}$$

Substituting (23a)–(23c) into (22), we obtain

$$ {}_N\langle\psi_j|(\Delta W_i)^2|\psi_j\rangle_N - \tfrac{1}{4}\,{}_N\langle\psi_j|[A_-^M, A_+^M]|\psi_j\rangle_N = 0 \qquad (i = 1, 2) \tag{24}$$

This indicates that the N states of (9) are all minimum-uncertainty states of the operators $W_1(M)$ and $W_2(M)$ ($M = nN$, $n = 1, 2, \ldots$) defined by (19).

3.2.2. $M = nN + i$ ($n = 0, 1, 2, \ldots;\ i = 1, 2, \ldots, N - 1$) for Odd N

In these conditions, for all N states of (9), we have

$$ {}_N\langle\psi_j|A_+^{2M}|\psi_j\rangle_N = {}_N\langle\psi_j|A_-^{2M}|\psi_j\rangle_N = {}_N\langle\psi_j|A_+^M|\psi_j\rangle_N = {}_N\langle\psi_j|A_-^M|\psi_j\rangle_N = 0 \tag{25}$$

Using relation (13), we obtain

$$ {}_N\langle\psi_S|A_+^M A_-^M|\psi_S\rangle_N = r^{2(nN+i)}\, B_{N-i+S}/B_S \qquad (S = 0, 1, 2, \ldots, i-1) \tag{26}$$

$$ {}_N\langle\psi_t|A_+^M A_-^M|\psi_t\rangle_N = r^{2(nN+i)}\, B_{t-i}/B_t \qquad (t = i, i, + 1 \ldots, N - 1) \tag{27}$$

Therefore, for the states $|\Psi_S\rangle_N$ ($S = 0, 1, \ldots, i - 1$) and $|\Psi_t\rangle_N$ ($t = i, i + 1, \ldots, N - 1$), we have

$$ {}_N\langle\Psi_S|(\Delta W_1)^2|\Psi_S\rangle_N - \tfrac{1}{4}\,{}_N\langle\Psi_S|[A_-^M, A_+^M]|\Psi_S\rangle_N = \tfrac{1}{2} r^{2(nN+i)}\, B_{N-i+S}/B_S \tag{28}$$

$$ {}_N\langle\Psi_t|(\Delta W_1)^2|\Psi_t\rangle_N - \tfrac{1}{4}\,{}_N\langle\Psi_t|[A_-^M, A_+^M]|\Psi_t\rangle_N = \tfrac{1}{2} r^{2(nN+i)}\, B_{t-i}/B_t \tag{29}$$

From (10), we have $B_j(r^2) > 0$ when $r = |\beta| \neq 0$. The right-hand sides of (28) and (29) are larger than zero. Therefore, none of the N states given by (9) has Mth-order ($M = nN + i$; $n = 0, 1, 2, \ldots$; $i = 1, 2, \ldots, N - 1$) $SU(1, 1)$ squeezing in these conditions.

3.2.3. $M = nN + i$ ($n = 0, 1, \ldots, i = 1, 2, \ldots, N/2 - 1, N/2 + 1, \ldots, N - 1$) for Even N

With the above discussion, it can be proved that under these conditions, none of the N states given by (9) has Mth-order $SU(1, 1)$ squeezing.

3.2.4. $M = (n + 1/2)N$ ($n = 0, 1, 2, \ldots$) for Even N

Now we have

$$ \begin{aligned} {}_N\langle\psi_j|A_+^{2M}|\psi_j\rangle_N &= r^{(2n+1)N}\, e^{-i(2n+1)N\theta}, \\ {}_N\langle\psi_j|A^{2M}|\psi_j\rangle_N &= r^{(2n+1)N}\, e^{i(2n+1)N\theta} \end{aligned} \tag{30a}$$

$$ {}_N\langle\psi_j|A_+^M|\psi_j\rangle_N = {}_N\langle\psi_j|A_-^M|\psi_j\rangle_N = 0 \tag{30b}$$

Making use of (13), we obtain

$$
{}_N\langle\psi_S|A_+^M A_-^M|\psi_S\rangle_N = r^{(2n+1)N}\, B_{N/2+S}/B_S \quad (S = 0, 1, 2, \ldots, N/2 - 1) \tag{31}
$$

$$
{}_N\langle\psi_t|A_+^M A_-^M|\psi_t\rangle_N = r^{(2n+1)N}\, B_{t-N/2}/B_t \quad (t = N/2, N/2 + 1, \ldots, N - 1) \tag{32}
$$

Consequently, for the states $|\Psi_S\rangle_N$ $(S = 0, 1, \ldots, N/2 - 1)$ and $|\Psi_t\rangle_N$ $(t = N/2, \mathrm{N}/2 + 1, \ldots, N - 1)$ we have, respectively,

$$
\begin{aligned}
&{}_N\langle\psi_S|(\Delta W_1)^2|\psi_S\rangle_N - \tfrac{1}{4}\,{}_N\langle\psi_S|[A_-^M, A_+^M]|\psi_S\rangle_N \\
&\quad = \tfrac{1}{2} r^{(2n+1)N}[B_{N/2+S}/B_S + \cos(2n + 1)N\theta]
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
&{}_N\langle\psi_t|(\Delta W_1)^2|\psi_t\rangle_N - \tfrac{1}{4}\,{}_N\langle\psi_t|[A_-^M, A_+^M]|\psi_t\rangle_N \\
&\quad = \tfrac{1}{2} r^{(2n+1)N}[B_{t-N/2}/B_t - \cos(2n + 1)N\theta]
\end{aligned} \tag{34}
$$

According to (33) and (34), the conditions which ensure the existence of Mth-order $[M = (n + 1/2)N,\ n = 0, 1, 2, \ldots]$ $SU(1, 1)$ squeezing in the states $|\Psi_S\rangle_N$ $(S = 0, 1, 2, \ldots, N/2 - 1)$ and $|\Psi_t\rangle_N$ $(t = N/2, N/2 + 1, \ldots, N - 1)$ are, respectively,

$$
B_{N/2+\mathrm{S}}/B_S + \cos(2n + 1)N\theta < 0 \tag{35}
$$

$$
B_{t-N/2+}/B_t - \cos(2n + 1)N\theta < 0 \tag{36}
$$

Since $B_j(r^2) > 0$ when $r = |\beta| \neq 0$, it can be seen that if N and n are given, (35) or (36) will be satisfied provided that the modulus r and argument θ of the complex parameter β are chosen properly. Therefore, for even N there exists Mth-order $[M = (n + 1/2)N;\ n = 0, 1, 2, \ldots]$ $SU(1, 1)$ squeezing in the eigenstates of the operator A^N.

4. ANTICORRELATION IN THE EIGENSTATES OF THE OPERATOR A^N

Now, for the N states $|\psi_j\rangle_N$ given by (9), we consider the second-order correlation function defined by [12]

$$
g_{2,j} = \frac{{}_N\langle\psi_j|A_+^2 A_-^2|\psi_j\rangle_N}{{}_N\langle\psi_j|A_+ A_-|\psi_j\rangle_N^2} \qquad (j = 0, 1, 2, \ldots, N - 1) \tag{37}
$$

If $g_{2,j} < 1$, we say that there is anticorrelation [12] in the states $|\psi_j\rangle_N$. This kind of characteristic is formally similar to the antibunching effect of a light field [17]. We study this anticorrelation in the N states given by (9).

Using (13) and (37), for the N states given by (9), we obtain

$$g_{2,0} = \frac{{}_N\langle\psi_0|A_+^2 A_-^2|\psi_0\rangle_N}{{}_N\langle\psi_0|A_+ A_-|\psi_0\rangle_N^2} = \frac{B_0 B_{N-2}}{B_{N-1}^2} \tag{38}$$

$$g_{2,1} = \frac{{}_N\langle\psi_1|A_+^2 A_-^2|\psi_1\rangle_N}{{}_N\langle\psi_1|A_+ A_-|\psi_1\rangle_N^2} = \frac{B_1 B_{N-1}}{B_0^2} \tag{39}$$

$$g_{2,j} = \frac{{}_N\langle\psi_j|A_+^2 A_-^2|\psi_j\rangle_N}{{}_N\langle\psi_j|A_+ A_-|\psi_j\rangle_N^2} = \frac{B_{j-2} B_j}{B_{j-1}^2} \qquad (j = 2, 3, \ldots, N-1) \tag{40}$$

Substituting (10) into (38), we obtain

$$\begin{aligned} g_{2,0} = {} & \sum_{m=0}^{\infty} \left\{ \sum_{n=0}^{m} [(nN)!(2k)_{nN}(mN - nN + N - 2)!(2k)_{mN-nN+N-2}]^{-1} \right\} y^{mN} \\ & \times \left(y^N \sum_{m=0}^{\infty} \left\{ \sum_{n=0}^{m} [(nN + N - 1)!(2k)_{nN+N-1} \right.\right. \\ & \left.\left. (mN - nN + N - 1)!(2k)_{mN-nN+N-1}]^{-1} \right\} y^{mN} \right)^{-1} \\ = {} & f_1(y)/[y^N f_2(y)] \end{aligned} \tag{41}$$

where $y = r^2 = |\beta|^2$. For $N \geq 3$, we have

$$\begin{aligned} & \sum_{n=0}^{m} \frac{1}{(nN)!(2k)_{nN}(mN - nN + N - 2)!(2k)_{mN-nN+N-2}} \\ & > \sum_{n=0}^{m} \frac{1}{(nN + N - 1)!(2k)_{nN+N-1}(mN - nN + N - 1)!(2k)_{mN-nN+N-1}} \end{aligned} \tag{42}$$

and hence $f_1(y) > f_2(y)$, so that $g_{2,0} > 1$ when $y < 1$. However, when $y > 1$, there surely exist values of y [e.g., $y^N > f_1(y)/f_2(y)$] for which the following relation holds:

$$g_{2,0} = f_1(y)/[y^N f_2(y)] < 1 \tag{43}$$

Substituting (10) into (39), we have

$$\begin{aligned} g_{2,1} &= \frac{y^N \sum_{m=0}^{\infty} \left\{ \sum_{n=0}^{m} [(nN + 1)!(2k)_{nN+1}(mN - nN + N - 1)!(2k)_{mN-nN+N-1}]^{-1} \right\} y^{mN}}{\sum_{m=0}^{\infty} \left\{ \sum_{n=0}^{m} [(nN)!(2k)_{nN}(mN - nN)!(2k)_{mN-nN}]^{-1} \right\} y^{mN}} \\ &= y^N f_3(y)/f_4(y) \end{aligned}$$

Obviously,

$$\sum_{n=0}^{m} \frac{1}{(nN+1)!(2k)_{nN+1}(mN-nN+N-1)!(2k)_{mN-nN+N-1}} < \sum_{n=0}^{m} \frac{1}{(nN)!(2k)_{nN}(mN-nN)!(2k)_{mN-nN}} \tag{45}$$

so that $f_3(y) < f_4(y)$. Therefore $g_{2,1} < 1$ when $y^N < f_4(y)/f_3(y)$.

From (10) and (40), we obtain

$$g_{2,j} = \frac{\sum_{m=0}^{\infty}\left\{\sum_{n=0}^{m}[(nN+j-2)!(2k)_{nN+j-2}(mN-nN+j)!(2k)_{mN-nN+j}]^{-1}\right\}y^{mN}}{\sum_{m=0}^{\infty}\left\{\sum_{n=0}^{m}[(nN+j-1)!(2k)_{nN+j-1}(mN-nN+j-1)!(2k)_{mN-nN+j-1}]^{-1}\right\}y^{mN}}$$

$$< \frac{\sum_{m=0}^{\infty}[(m+1)/(j-2)!(2k)_{j-2}j!(2k)_j]y^{mN}}{\sum_{m=0}^{\infty}(m+1)/[(mN+j-1)!(2k)_{mN+j-1}]^2}$$

$$< \frac{[j!(2k)_j(j-2)!(2k)_{j-2}]^{-1}\sum_{m=0}^{\infty}(m+1)y^{mN}}{[(j-1)!(2k)_{j-1}]^{-2}} \qquad (j = 2, 3, \ldots, N-1)$$

Obviously,

$$\lim_{y\to 0}\sum_{m=0}^{\infty}(m+1)y^{mN} = 1 \tag{47}$$

Therefore, from (46), we obtain

$$\lim_{y\to 0} g_{2,j} < \frac{[(j-1)!(2k)_{j-1}]^2}{j!(2k)_j(j-2)!(2k)_{j-2}} = \frac{(j-1)(2k+j-2)}{j(2k+j-1)} < 1, \qquad (j = 2, 3, \ldots, N-1) \tag{48}$$

Therefore, there is anticorrelation in the states $|\psi_j\rangle_N$ $(j = 2, 3, \ldots, N-1)$ when $y \to 0$.

We sum up the above results and obtain that, in different ranges of $y = |\beta|^2$, there is anticorrelation in all of the N states given by (9).

5. CONCLUSION

In this paper, the N eigenstates of the higher powers A^N $(N \geq 3)$ of the annihilation operator of an anharmonic oscillator are constructed. We defined higher order $SU(1, 1)$ squeezing and studied its properties and anticorrelation of the N states. For the N eigenstates of the operator A^N, we come to the following conclusions: (a) Their linear combination may form a complete representation. (b) For odd N, none of them has higher order $SU(1, 1)$ squeezing. (c) For odd N and even N, all of them are minimum-uncertainty states of the operators $W_1(M)$ and $W_2(M)$ ($M = nM$, $n = 0, 1, 2, \ldots$) defined by (19). (d) For even N, when $M = (n + 1/2)N$ ($n = 0, 1, 2, \ldots$), for all of them Mth-order $SU(1,1)$ squeezing exists, (e) There is anticorrelation in all N states.

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REFERENCES

1. R. J. Glauber (1963). *Phys. Rev.* **131**, 2766.
2. J. R. Klauber and B. S. Skagerstam, *Coherent States*, World Scientific, Singapore (1985).
3. M. Hillery (1987). *Phys. Rev. A* **36**, 3796.
4. J. Z. Sun and J. S.Wang, *et al.* (1991). *Phys. Rev. A* **44**, 3369.
5. J. Z. Sun and J. S. Wang, *et al.* (1992). *Phys. Rev. A* **46**, 1700.
6. J. Z. Sun and J. S. Wang, *et al.* (1991). *J. Mod. Opt.* **38**, 2295.
7. Z. W. Xu (1996). *Acta Physica Sinica* **45**, 1807 [in Chinese].
8. Z. X. Yu and J. S. Wang, *et al.* (1997). *Acta Physica Sinica* **46**, 1693 [in Chinese].
9. J. L. Zhang and Q. Z. Guo (2000). *Acta Sinica Quantum Optics* **6**, 18 [in Chinese].
10. D. P. Zhu (1987). *J. Phys. A* **20**, 4331.
11. M. M. Nieto and L. M. Simmons Jr. (1979). *Phys. Rev. D* **20**, 1321.
12. M. Ban (1994). *Phys. Lett. A* **193**, 121.
13. A. O. Barut and L. Girardello (1971). *Commun. Math Phys.* **21**, 41.
14. V. Bužek (1990). *J. Mod. Opt.* **37**, 303.
15. S. R. Hao (1993). *Acta Physica Sinica* **42**, 1057 [in Chinese].
16. Z. M. Zhang , L. Xu and F. L. Li (1990). *Phys. Lett. A* **150**, 27.
17. D. F. Walls (1983). *Nature* **306**, 141.